Agricultural Systems **41** (1993) 257–274

A Fuzzy Multiple Objective Linear Programming Approach to Forest Planning Under Uncertainty

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ABSTRACT

Describes the use of Fuzzy Multiple Objective Linear Programming (FMOLP) in forest planning where imprecise objective function coefficients are present. An extended formulation is also described for planning situations where uncertainties occur in the constraint set. A sample problem is presented to illustrate the approach.

INTRODUCTION

During the last two decades, mathematical programming models ha been used extensively in forest planning, with linear programming (L being the most commonly used method. However, concerns about t use of LP models have also been raised (Bare & Field, 1987). The ma criticisms deal with the inherently deterministic nature of LP models, a their use of precise coefficients. In traditional LP models, the coefficien or parameters are assumed to be known with certainty. In many re world forest planning problems, however, it is very unlikely that this a sumption is valid. For example, forest managers often have to deal wi insufficient or imperfect information due to the inherent complexity the system (Allen & Gould, 1986). Hence, to enhance model utility, it necessary to be able to incorporate imprecise or uncertain informati into the model.

The term 'uncertainty' has been widely used to denote several phenomen

Agricultural Systems 0308-521X/93/$06.00
Printed in Great Britain

has been used to represent risks, imprecision, randomness, inaccuracy, ıbiguity or inexactness. In this paper, uncertainty is used to reflect any enomena other than those regarded as random or probabilistic in nature.

There are several reasons for incorporating uncertainty in forest planıg. First, forest planning involves long planning horizons (e.g. several cades). Accurate long-term projections are generally difficult to make d are at best only educated guesses of future outcomes. Future timber ices, for instance, are highly dependent on several variables making em difficult to predict. Moreover, most forest lands covering large dirse geographical areas produce multiple goods and services which are lued differently by forest users. Some of these uses can be adequately easured while others are inherently qualitative and difficult to quantify. nally, forest planning often requires the incorporation of human subtivity which is both difficult to elicit and express in quantitative terms. ierefore, the use of optimization models that can incorporate imprecise ormation, has become a prerequisite to comprehensive planning, parularly in complex planning environments, such as forestry.

Several methods have been suggested to deal with imprecision and certainty in forest planning. One such method is parametric linear ogramming (Navon & McConnen, 1967; Weintraub & Ingram, 1981; redes & Brodie, 1988). This approach can be used to examine the anges in the LP solution as one or more parameters — usually in some stematic or fixed proportion — are changed over a wide range of lues. However, as Pickens & Dress (1988) point out, this approach es not seem to be a viable approach for large-scale forestry problems. ıother suggested method is probabilistic or stochastic programming hompson & Haynes, 1971; Hunter *et al.*, 1976; Hof *et al.*, 1988; Picks & Dress, 1988). For certain types of problems where uncertainty is ainly due to randomness, these probability-based methods are approate. However, for other uncertain-ties (e.g. imprecision, ambiguity, inactness and inaccuracies) these stochastic models may not be as ective and efficient. A relatively new approach called fuzzy programng may be better suited under these environments. The purpose of this per is to develop a fuzzy multiple objective linear programming model forest planning that accommodates imprecise information. The paper organized as follows; first, a single objective function with intervallued coefficients is formulated as a two-objective function problem. en, in the presence of multiple objectives, some of which have exact efficients while others have interval-valued coefficients, the problem is rmulated as a multiple objective linear programming problem. Finally, uzzy multiple objective linear programming model is formulated with th interval-valued and exact coefficients.

BACKGROUND

The background of the fuzzy approach for forest planning is found i the literature on fuzzy sets and fuzzy linear programming (FLP). A bri discussion on some FLP concepts is provided in this section but, f more details, readers are referred to Zadeh (1965), Dubois & Prad (1980), and Zimmermann (1985, 1987). Bellman & Zadeh (1970) pr vided the seminal work on decision making in a fuzzy environment an developed the original methodological basis for the development of fuz mathematical programming methods. Since then, a number of alternati methodologies have been proposed. Most notable among these are tho described by: Zimmermann (1975, 1978), Narasimham (1980), Hanna (1981), Chanas (1983), Chanas & Kulej (1984), Tanaka *et al.* (198 1985), Verdegay (1984), Orlovski (1984), Tiwari *et al.* (1987), Delgado *al.* (1989), and Rommelfanger *et al.* (1989).

A convenient way to describe FLP is to begin with the convention linear programming problem;

$$\left.\begin{aligned} \text{Max } Z &= CX \\ AX &\leq \text{B} \\ X &\geq 0 \end{aligned}\right] \qquad (\;$$

where A is an (m $\times$ n) matrix, $C \in R^n$ is a row vector and $X \in R^n$, and $\in R^m$ are column vectors. Consider the objective function coefficients co tained in C. Rather than exact values, assume that the decision mak (DM) only can provide approximate estimates of the values of the obje tive function coefficients. Several authors have suggested ways to de with this problem. Most of them are based on the concepts of fuz numbers and parameters. For example, Orlovski (1984, 1985) shows line objective functions with fuzzy parameters, while Tanaka *et al.* (1984) e amines linear programming with triangular fuzzy numbers. Tanaka *et* (1985) and Delgado *et al.* (1987) also describe linear programming wi trapezoid fuzzy parameters to represent imprecise objective coefficients.

In this paper, it is assumed that the DM only can specify the coef cients in the objective function as intervals $[C_i^l, C_i^u]$, $i = 1,2,\ldots,n$, rath than exact values. Furthermore, these interval coefficients are themselv derived from interval estimates provided by the DM. Hence, in order better understand the interval-valued objective function coefficients the model proposed in this paper, it is necessary to describe some co cepts concerning interval estimation and optimization. According Moor (1966, 1969) and Kaufmann & Gupta (1988), an interval numb is defined to be an ordered pair of real numbers, $[a, b]$, with $a \leq b$. $[a,$ is the set of real numbers y such that $a \leq y \leq b$ or $[a, b] = \{y \mid a \leq y \leq b$

Arithmetic operations with intervals are defined as follows:

$$[a, b] + [c, d] = [a + c, b + d], \tag{2}$$

$$[a, b] - [c, d] = [a - d, b - c], \tag{3}$$

$$[a, b]\,[c, d] = [\min(ac, ad, bc, bd), \max(ac, ad, bc, bd)], \tag{4}$$

$$[a, b] / [c, d] = [a, b]\,[1/d, 1/c] \text{ (if } 0 \in [c, d]) \tag{5}$$

In the traditional LP problem described in eqn (1) a unique objective nction is defined for every set of objective function coefficients. However, if these coefficients are expressed as intervals, the problem expands om a single objective problem to a problem that contains an infinite mber of objective functions for all $x \in X = \{X \in R^n \mid AX \leq B, X \geq 0\}$. other words, the problem expands to take into account all vectors $C \in$ of the bounded interval $C^\circ = \{C \mid C^l \leq C \leq C^u\}$ as parameters.

Bitran (1980) examines linear multiple objective problems with interval efficients and suggests a possible solution approach by obtaining a subpblem that generates and tests if a feasible extreme point solution is icient (i.e. there exists no other solution that can bring improvement in least one objective without degrading other objectives). On the other nd, Rommelfanger *et al.* (1989) proposes an approach which involves selection of a single representative C_i in each interval $[C_i^l, C_i^u]$, and en solves the following LP problem;

$$\text{Max } \{CX \mid AX \geq B, X \geq 0\}$$

ng conventional LP algorithms. Following the concept of Bierman *al.* (1986) and Render & Stair (1988), Rommelfanger *et al.* (1989) ggest several forms of the objective function. If the DM is optimistic, en one may choose the 'upper side' of the objective function. That is, objective function Z is of the form;

$$Z^u = C^u X \tag{6}$$

contrast, if one is pessimistic then the 'lower side' of the objective nction may be selected, which is of the form;

$$Z^l = C^l X \tag{7}$$

risk neutral DM may elect to use

$$Z = (Z^l + Z^u)/2 = (1/2)\,(C^l + C^u)X \tag{8}$$

More generally, the objective function may be expressed as;

$$\begin{aligned} Z &= (1 - \alpha)\, Z^l + \alpha Z^u \\ &= \{(1 - \alpha)\, C^l + \alpha C^u\}X \qquad 0 \leq \alpha \leq 1 \end{aligned} \tag{9}$$

where α is called the optimism/pessimism parameter. Intuitively, eqn (9) h the advantages of allowing the DM to incorporate personal feelings ar prior knowledge into the problem. However, the disadvantage of the metho is that it requires information from the DM which may not be known.

If the DM feels uncomfortable in choosing a suitable objective fun tion, a compromise objective function may be chosen by progressive r duction of the objective space (i.e. $C \in R^n$) as proposed by Rommelfang *et al.* (1989). This approach reduces many objective functions into:

$$\text{Max } Z = CX \quad \text{with } C \in C^{\circ}$$

by extremely positioning the two objective functions; Max Z^l and Ma Z^u, and finding the solution of the following vector-optimization problem

$$\left[\begin{array}{ll} & \text{Max}\begin{bmatrix} Z^l \\ Z^u \end{bmatrix} = \text{Max}\begin{bmatrix} C^l x \\ C^u x \end{bmatrix} \\ \text{subject to} & \\ & AX \leq B \\ & X \geq 0 \end{array}\right] \quad (1$$

Note, the single objective LP problem is converted into a two objecti problem. This approach is more flexible and allows the generation of compromise solution within the interval denoted by the two objective C^lX and C^uX. Solving the two-objective problem, including addition objectives whose coefficients may be exact or interval-valued, requires t use of multiple objective programming techniques.

FUZZY MULTIPLE OBJECTIVE LINEAR PROGRAMMING (FMOLP)

Consider a multiple objective problem;

$$\left[\begin{array}{ll} \text{Max} & \begin{bmatrix} Z_1 = C_1X \\ Z_2 = C_2X \\ \\ Z_k = C_kX \end{bmatrix} \\ \text{subject to} & \\ & AX \leq B \\ & X \geq 0 \end{array}\right] \quad (1$$

Among the objective functions, some have unique coefficients, b others are more loosely defined and with imprecise coefficients. For t latter case, the coefficients are represented by interval values instead act values. Furthermore, the membership functions of the objectives e as follows:

$$u_i(X) = \begin{cases} 0 & \text{if } z_i(X) \le f_{1i} \\ \dfrac{z_i(X) - f_{1i}}{f_{0i} - f_{1i}} & \text{if } f_{1i} \le z_i(X) \le f_{0i} \\ 1 & \text{if } f_{0i} \le z_i(X) \end{cases} \tag{12}$$

here f_{0i} is the optimal or most desirable value for objective i, and f_{1i} is e least desirable or tolerant value for objective i.

The membership function is one of the basic tenets of fuzzy set theory. is used as a primary instrument to incorporate inexactness into formal otimization procedures. Zimmermann (1987) and Mendoza & Sprouse 989) provide overviews of the role of membership functions in fuzzy ecision making.

An intuitive explanation of the membership function in the context of ecision making is as follows: the decision maker is very satisfied (i.e. the embership function or degree of satisfaction is equal to 1) if a solution yields an objective function value at least equal to f_{0i}; he is less satisfied ith a solution that gives an objective function value less than f_{0i}; and he completely unsatisfied (i.e. the degree of satisfaction is equal to 0) if a lution X yields an objective function value less than f_{1i}.

The problem now is to simultaneously satisfy all objective functions rep- sented by their corresponding membership functions. Each objective hose coefficients are expressed as interval values is represented as two ojective functions in eqn (11), following the extreme positioning concept described in eqn (10). Thus, following the fuzzy approach described by mmermann (1978); the FMOLP model can be formulated as follows:

Max Θ

subject to

$$\Theta \le \frac{z_i(X) - f_{1i}}{f_{0i} - f_{1i}} \qquad i = 1, 2, \ldots, k \tag{13}$$

$$AX \le B$$

$$X \ge 0 \qquad \Theta \ge 0$$

The formulation above follows the MAXMIN approach where the ob- ctive is to find a solution that yields the maximum membership func- on value, Θ, which satisfies the constraint described in eqn (13). That is, is the highest minimum degree of satisfaction considering all objectives d their respective desirable limits denoted by f_{0i} and f_{1i}. Some implica- ons of the FMOLP formulation are described in the next section using e results from the sample problem.

A CASE STUDY

To illustrate the FMOLP model described in eqn (13), a sample proble adopted from Johnson *et al.* (1986) is used. (For details, please refer Johnson *et al.* (1986) and Johnson & Crim (1986)). The sample proble was modified to reflect multiple objectives as previously described in Ba & Mendoza (1988). The sample forest is the Brush Mountain Nation Forest located at the western slopes of the Appalachians in We Virginia. The forest contains loblolly pines in two age classes, mix hardwoods in one age class, and a meadow. Each age class comprised a number of individual stands which were grouped into four analy areas. Table 1 gives the multiobjective programming formulation of t problem with four objective functions optimized over three 10-ye periods. For illustrative purposes, three of the objective functio namely, sediment, timber, and forage, have exact coefficients while the c efficients of the net present value (NPV) objective function are represent as interval. The sediment objective is also represented as a constrai requiring that the maximum allowable amount of sediment is 4200 tons

Based on the yields, costs and interest rates, the NPVs are comput using the arithmetic operations described in eqns (2)–(5). Tables 2, 3, and 5 give details of how the interval values are calculated. To illustra the computational procedure, an example for analysis area #2 (mix hardwood) is presented. In period 3, the forest becomes 40 years c (Table 2). The timber yield for the regeneration harvest is estimated to 1 050 (cu ft/acre). The stumpage value is assumed to be within the inte val [0·165, 0·245] ($/cu ft). The total revenue per acre is expressed as t interval 1050* (0·165, 0·245) = [173·25, 257·25] ($/acre). From Table the road construction cost is between [81,99] ($/acre) while the layo and sale cost is [0·07, 0·13] ($/cu ft) or [73·5, 136·5] ($/acre). The to costs are, therefore [81, 99] + [73·5, 136·5] = [154·5, 235·5] ($/acre), a the net revenue is [173·25, 257·25] – [154·5, 235·5] = [–62·25, 103] ($/acr In Table 5, the discount factor $(1+r)^n$ where r is expressed as an interv is between [2·3068, 3·0782] when n is 25. Therefore, the NPV for this a tivity is the net revenue $(1+r)^n$ = [–62·25, 103]/ [2·3068, 3·0782] = [−26·9 44·65] ($/acre). Similarly, the NPV for overwood removal is calculated (–1·64, 34·52) ($/acre). Finally, the total NPV from the harvests of t mixed hardwood area are computed as; [–26·99, 44·65] + [−1·64, 34·5] [–28·63, 79·17] ($/acre).

The coefficients for all other variables associated with analysis un 1–3 are similarly computed and are shown in Tables 2, 3, 4, and 5. F analysis area 4, the NPVs are estimated to be [–24, –12], (–34, 10), a (–43, 7), for the variables X_{4L1}, X_{4H1} and X_{4H2}, respectively.

A Linear Programming Formulation of the Sample Problem with Four Objective Functions

	Analysis area #1 loblolly pine age 5			Analysis area #2 mixed hardwood age 15				Analysis area #3 loblolly pine age 15							Analysis area #4 meadow			Accounting variable				
	x_{1T1}	x_{1T2}	x_{1M1}	x_{2T1}	x_{2T2}	x_{2T3}	x_{2M1}	x_{3T1}	x_{3T2}	x_{3T3}	x_{3M1}	x_{3B1}	x_{3B2}	x_{3C1}	x_{4L1}	x_{4H2}	x_{4H2}	H_1	H_2	H_3	C	
Max (NPV)	(36, 118)	(110, 274)		(–79, –33)	(–54, 14)	(–29, 79)		(486, 600)	(313, 514)	(184, 435)		(129, 256)	(107, 221)		(–24, –12)	(–34, 10)	(–43, 12)					NPV
Min(sed)	2	4		4	6	8		3	3	3		1·4	0·7		3	4·54	4					sediment
Max(tim)	1 500	2 300		835	1 255	1 050		3 000	3 300	3 500		1 900	1 600									timber
Max(for)															200	300	300					forage
Subject to:																						
Land	1	1	1																			= 200 acres
				1	1	1	1															= 300 acres
								1	1	1	1	1	1	1								= 700 acres
															1	1	1					= 300 acres
Bird habitat												1	1	1								= 260 acres
Timber				490				3 000				900						–1				= 0
yield	1 500			345	805				3 300				1 600						–1			= 0
		2 300			450	1 050				1 500		1 000								–1		= 0
																		–1	1			≥ 0
																			–1	1		≥ 0
Sed const	2	4		4	6	8		3	3	3		1·4	0·7		3	4·5	4					≤ 4 200
Clear	1	1						1	1	1											–1	= 0
cut								1													–0·4	≤ 0
	1								1												–0·4	≤ 0
		1								1											–0·4	≤ 0

Decision variables: x_{sjk} = acres assigned to timing choice *k* of prescription *i* of analysis area *s* where *i* can be T (timber), M (minimum level), B (bird/timber), C (bird/minimum level), L (low intensity forage), and H (high intensity forage).

Accounting variables: H_j = total timber harvest volume in period *j*; C = acres assigned to prescription that allow clear cutting.

TABLE 2
NPV from Harvesting the Existing Timber Stands

Analysis area	*Period*	*Age (yrs)*	*Regeneration harvest*						*Overwood removal*						*Total NPV*
			Timber yield (ft^3/acre)	*Stumpage value (\$/ft^3)*	*Total revenue (\$/acre)*	*Total cost (\$/acre)*	*Net revenue (\$/acre)*	*NPV (\$/acre)*	*Timber yield (ft^3/acre)*	*Stumpage value (\$/ft^3)*	*Total revenue (\$/acre)*	*Total cost (\$/acre)*	*Net revenue (\$/acre)*	*NPV (\$/acre)*	*(\$/acre)*
#1 loblolly	1	10	500	(0·085, 0·115)	(43, 58)	(110, 121)	(−78, −52)	(−64·73, −42·33)							
pine age 5	2	20	1 500	(0·170, 0·230)	(255, 345)	(144, 186)	(69, 201)	(36·17, 118·25)							
	3	30	2 300	(0·255, 0·345)	(587, 794)	(161, 249)	(338, 632)	(109·80, 273·97)							
	4	40	3 000	(0·240, 0·360)	(720, 1 080)	(168, 312)	(408, 912)	(79·07, 302·84)							
#2	1	20	490	(0·105, 0·135)	(51, 66)	(131, 147)	(−95, −65)	(−79·21, −52·87)	345	(0·12, 0·18)	(41, 62)	(27, 41)	(0, 35)	(0, 20·30)	(−721, −32·57)
mixed	2	30	805	(0·120, 0·180)	(97, 145)	(148, 193)	(−96, −40)	(−56·48, −20·70)	450	(0·15, 0·25)	(68, 113)	(32, 59)	(9, 81)	(2·92, 35·11)	(−53·56, 14·40)
hardwood	3	40	1 050	(0·165, 0·245)	(173, 257)	(155, 236)	(−62, 103)	(−26·99, 44·65)	495	(0·13, 0·27)	(64, 134)	(30, 69)	(−5, 104)	(−1·64, 34·52)	(−28·63, 79·17)
age 15	4	50	1 155	(0·140, 0·260)	(162, 300)	(147, 264)	(−102, 153)	(−33·87, 50·81)							
#3	1	40	3 000	(0·285, 0·315)	(855, 945)	(222, 258)	(597, 723)	(486·00, 600·00)							
loblolly	2	50	3 300	(0·270, 0·330)	(891, 1 089)	(216, 294)	(597, 723)	(312·94, 513·59)							
pine	3	60	3 500	(0·255, 0·345)	(893, 1 207)	(204, 327)	(566, 1 004)	(183·87, 435·23)							
age 35	4	70	3 600	(0·240, 0·360)	(864, 1 260)	(186, 354)	(510, 1 074)	(98·84, 356·63)							

TABLE 3
Total Costs for the Sample Problem

nalysis rea	Age	*Road construction cost (\$/acre)*	*Layout & sale cost (\$/100 ft^3)*	*Layout & sale cost (\$/acre)*	*Total cost (\$/acre)*
1	10	(87, 93)	(4·5, 5·5)	(22·5, 27·5)	(109·5, 120·5)
	20	(84, 96)	(4, 6)	(60, 90)	(144, 186)
	30	(81, 99)	(3·5, 6·5)	(80·5, 149·5)	(161·5, 248·5)
	40	(78, 102)	(3, 7)	90, 210)	(168, 312)
2	20	(87, 93)	(9, 11)	(44·1, 53·9)	(131·1, 146·9)
	30	(84, 96)	(8, 12)	(64·4, 96·6)	(148·4, 192·6)
	40	(81, 99)	(7, 13)	(73·5, 136·5)	(154·5, 235·5)
	50	(78, 102)	(6, 14)	(69·3, 161·7)	(147·3, 263·7)
3	40	(87, 93)	(4·5, 5·5)	(135, 165)	(222, 258)
	50	(84, 96)	(4, 6)	(132, 198)	(216, 294)
	60	(81,99)	(3·5, 6·5)	(122·5, 227·5)	(203·5, 326·5)
	70	(78, 102)	(3, 7)	(108, 252)	(186, 354)
2 overwood removal	30		(8, 12)	(27·6, 41·4)	(27·6, 41·4)
	40		(7, 13)	(31·5, 58·5)	(31·5, 58·5)
	50		(6, 14)	(29·7, 69·3)	(29·7, 69·3)

TABLE 4
NPV of Loblolly Pine Stands in Area #3 Under the Bird Habitat Prescription

Planning period	*Harvest volume (ft^3/acre)*	*Stumpage value (\$/ft^3)*	*Total revenue (\$/acre)*	*Total cost (\$/acre)*	*Net revenue (\$/acre)*	*Harvest NPV (\$/acre)*	*Cumulative NPV (\$/acre)*
rvest schedule #1							
1	900	(0·23, 0·27)	(207, 243)	(68, 83)	(125, 176)	(101·35, 145·64)	
2							
3	1000	(0·19, 0·31)	(190, 310)	(56, 104	(86, 254)	(27·94, 110·11)	(129·29, 255·75)
rvest schedule #2							
1							
2	1600	(0·21, 0·29)	(336, 464)	(88, 1320)	(204, 376)	(106·94, 221·20)	(106·94, 221·20)

TABLE 5
Discount Factors for the Sample Problem

Age	Period	n	r	$(1 + r)^n$
10	1	5	(0·038, 0·042)	(1·205 0, 1·228 4)
20	2	15	(0·036, 0·044)	(1·699 8, 1·907 7)
30	3	25	(0·034, 0·046)	(2·306 8, 3·078 2)
40	4	35	(0·032, 0·048)	(3·011 5, 5·159 9)

Following eqn (10), the NPV function is now positioned into two o jective functions. A multiple objective programming problem with fi objective functions is thus formulated using eqn (11) as follows,

$$\left.\begin{array}{ll}\left.\begin{array}{l}\text{Max } Z_1^{l} = C_1^{l}x \\ \text{Max } Z_1^{u} = C_1^{u}x\end{array}\right] & \ldots\ldots \text{NPV} \\ \text{Max } Z_2 = C_2x & \ldots\ldots\ldots \text{sediment} \\ \text{Max } Z_3 = C_3x & \ldots\ldots\ldots \text{timber} \\ \text{Max } Z_4 = C_4x & \ldots\ldots\ldots \text{forage} \\ \text{subject to} & \\ AX \leq B & \\ X \geq 0 & \end{array}\right] \quad (1$$

Among the five objective functions, one (i.e. sediment) is to be mi mized. The membership function for the sediment objective function formulated as;

$$u_i(X) = \begin{cases} 0 & \text{if } C_2X \geq f_{12} \\ \dfrac{f_{12} - C_2X}{f_{12} - f_{02}} & \text{if } f_{12} > C_2X \geq f_{02} \\ 1 & \text{if } f_{02} > C_2X \end{cases} \quad (1$$

where f_{12} and f_{02} are the maximum tolerable and minimum desira amounts of sediment.

Following eqn (13), the FMOLP model for the sample problem is fo mulated as a MAXMIN problem described below:

Max Θ
subject to

$$\left.\begin{array}{l} C_1^{l}X - \Theta\,(f^{l}_{01} - f^{l}_{11}) \geq f^{l}_{1} \\ C_1^{u}X - \Theta\,(f^{u}_{01} - f^{u}_{11}) \geq f^{u}_{11} \\ C_2X + \Theta\,(f_{12} - f_{02}) \leq f_{12} \\ C_3X - \Theta\,(f_{03} - f_{13}) \geq f_{13} \\ C_4X - \Theta\,(f_{04} - f_{14}) \geq f_{14} \\ AX \leq B \\ X \geq 0 \end{array}\right] \quad (1$$

To find a solution using this formulation, the f_{0i}'s and f_{1i}'s must known. These values may be specified by the DM, or some benchma information may be used if available. Otherwise, these values can computationally derived using a payoff table as illustrated in Table 6. L f_k, $k = 1,2,\ldots,5$, be the feasible ideal values for the following five problems:

$$\left.\begin{aligned} &\text{Max (min) } f_k(x) = C_k X \quad k = 1, 2, \ldots, 5 \\ &\text{subject to} \\ &AX \le B \\ &X \ge 0 \end{aligned}\right] \qquad (17)$$

In Table 6, z_{ij} is the value of the ith objective function when the jth jective is optimized.

$$f_{1i} = \left[\begin{array}{ll} \min \{z_{ij}\} & \text{if } i = 1, 2, 4, 5. \\ \max \{z_{ij}\} & \text{if } i = 3. \\ & \quad j = 1, 2, \ldots, 5. \end{array}\right.$$

Similarly

$$f_{0i} = \left[\begin{array}{ll} \max \{z_{ij}\} & \text{if } i = 1, 2, 4, 5. \\ \min \{z_{ij}\} & \text{if } i = 3. \end{array}\right.$$

By solving eqn (16), a compromise solution is found which is sum-rized below.

$$\begin{array}{ll} x_{1M1} = 200 & x_{2M1} = 300 \\ x_{3T1} = 126 & x_{3T2} = 84 \\ x_{3T3} = 104 & x_{3M1} = 125 \\ x_{3B1} = 127 & x_{3B2} = 133 \\ x_{4L1} = 126 & x_{4H2} = 174 \\ H_1 = 49\,2020 & C = 315 \\ H_2 = 49\,2024 & \text{all other } x_{ijk} = 0 \\ H_3 = 49\,2024 & \Theta = 0{\cdot}579 \end{array}$$

and the corresponding objective values are

$$\begin{array}{ll} z_1{}^l = 12\,6980 & z_1{}^u = 22\,6879 \\ z_2 = 2289 & z_3 = 1476 \\ z_4 = 7\,7372 & \end{array}$$

TABLE 6

Pay-off Table: f_{0i} and f_{1i} Computation Results

jective alues	*Max $Z_l{}^l$*	*Max $Z_l{}^u$*	*Max Z_2*	*Max Z_3*	*Max Z_4*	f_{0i}	f_{1i}
$Z_l{}^l$	217 031	210 799	–7 200	193 636	–12 000	217 030	–10 200
$Z_l{}^u$	368 671	372 200	–3 600	352 408	3 000	372 200	–3 600
Z_2	4 200	4 200	900	4 200	1 350	900	4 200
Z_3	2 452	2 418	0	2 549	0	2 549	0
Z_4	60 000	90 000	60 000	60 000	90 000	90 000	60 000

$Z_l{}^u$: NPV = Net present value ($)
SED = Sediment yield (ton)
TBR = Timber production (MCF)
FOR = Forage production (AUM)

The results as described above show $\Theta = 0{\cdot}579$. This suggests that t highest degree (between 0 and 1) that the desirable levels (i.e. f_{0i}) can met simultaneously is 0·579. Comparing the actual objective function va ues above and the desirable and tolerable levels contained in Table 6, t membership function for each objective is calculated as follows: Z_1^l 0·603, $Z_1^u = 0{\cdot}613$, $Z_2 = 0{\cdot}579$, $Z_3 = 0{\cdot}579$, $Z_4 = 0{\cdot}579$.

The membership function Θ from the solution described above hel illustrate the meaning and implication of the FMOLP model described eqn (13). The membership function is a measure of the degree of satisfa tion of any solution. For a given objective, the target levels are specifi as an interval, a tolerable limit, f_{1i}, and a desirable limit f_{0i}. The line membership function in eqn (12) is formulated so that the membersh function for any objective i is equal to 1, if the desirable target is a tained; equal to 0, if objective i is achieved at the level below the tole able limit; and between 0 and 1, if the objective is attained at a val between f_{0i} and f_{1i}.

Any solution to eqn (13) yields different membership function valu (i.e. degree of satisfaction) for each objective. Some solutions will yie high membership functions for some objectives, and low values for oth objectives. The problem then is to choose the 'best' compromise soluti considering all membership function values of each objective. While may not be obvious from eqn (13), the FMOLP model is designed search for a solution that yields the highest minimum membership fun tion value (i.e. Max(Min $u_i(X)$) for all i). Intuitively, this implies a co promise solution where the objectives are at a minimum overall degree satisfaction equal to Θ. In the sample problem, the minimum degree satisfaction is 0·579. Except for the NPV objective, all the other three o jectives have degrees of satisfaction equal to 0·579.

Like any mathematical planning model, the solution generated abo represents only one out of a potentially large number of solutions. Usi sensitivity analysis, or the methods described by Mendoza & Sprou (1989), other solutions could be generated. In evaluating alternative sol tions generated by the FMOLP model, one measure of solution desirab ity is the actual value of Θ. Obviously, higher values of Θ are preferabl However, Θ is dependent on the specified target levels (f_{0i} and f_{1i}) so should be used with caution.

In some planning problems, objectives might be ranked or prioritize so that some objectives are valued more than others. Under this sit ation, other methods of combining the membership functions in eqn (1 could be used. As eqn (13) implies, all objectives are treated equall Mendoza & Sprouse (1989) offer some alternative approaches when o jectives are considered of unequal importance.

tensions of FMOLP model

e FMOLP model described above is formulated to accommodate im-
ecision only in the objective function coefficients. However, imprecision the constraint set is also pervasive in forest planning. For instance, ld coefficients typically used in growth projection and harvest schedul- models are subject to error and could possibly give inaccurate growth imates. Hence, constraints such as even flow or nondeclining yield mmonly used in US national forest planning, should also reflect these ccuracies in yield prediction. While this situation is not illustrated in sample problem, the FMOLP model can be generalized to accommo- te imprecision in the constraint set.

One way to model this situation is not to require that $AX \leq B$ in eqn 3) be strictly satisfied within the bounds specified by B. Instead, a cer- n amount of violation is tolerable. Following eqn (12), the member- p function of fuzzy constraints can also be described as;

$$u_i(X) = \begin{cases} 0 & \text{if } b'_i + p_i \leq (AX)_i \\ \dfrac{b'_i + p_i - (AX)_i}{p_i} & \text{if } b'_i < (AX)_i \leq b'_i + p_i \\ 1 & \text{if } (AX)_i < b'_i \end{cases} \tag{18}$$

ere p_i is the admissible tolerance in constraint i.

Hence, the general optimization problem that accommodates fuzziness the objective function and constraints can be formulated as,

$$\left.\begin{array}{l} \text{Max } \Theta \\ \text{subject to} \\ \qquad -\Theta(F_0 - F_1) + CX \geq F_1 \\ \qquad \Theta P + AX \leq B' + P \\ \qquad X \geq 0 \quad \Theta \geq 0 \end{array}\right] \tag{19}$$

ere F_0, F_1 are the desirable and least desirable targets for the objective nctions and B' and P are the tolerable limits and allowable deviation r all fuzzy constraints.

The generalized fuzzy formulation described in eqn (19) exhibits rtain characteristics that resemble goal programming. The similarities d differences between these two approaches are described elsewhere larasimhan, 1980, 1981; Hannan, 1981, 1982; Ignizio, 1982; Tiwari *et* , 1987).

SUMMARY

Uncertainty in forest planning is pervasive, entering in the form of a la
of information, imprecision or inaccuracies in estimating model para
eters, and inexact or imperfect data. All of these cause uncertainties th
must be incorporated in any planning model. For these kinds of unc
tainties, fuzzy programming approaches offer a convenient framewo
for planning and decision making.

Besides imprecision, forest planning is also inherently multiple obj
tive, mainly due to the multiple use nature of forest management. Hen
forest planning models should also address multiple objective concerns
forest management.

The two characteristics of forest management described above ma
forest management an appropriate environment for fuzzy multiple obj
tive programming models. In this paper, the FMOLP model develop
treats imprecision by specifying the objective function coefficients as i
terval values, instead of exact numbers. In addition, target values f
each objective representing desirable and least desirable limits, are al
specified. The model is applied to a sample problem where stumpa
prices, costs, and interest rates are specified as intervals resulting in int
val-valued coefficients of the NPV objective function. The three remai
ing objectives are assumed to be precise or deterministic, although th
could have been treated as interval valued.

The FMOLP model developed in this paper follows the 'extreme po
tioning' concept proposed by Rommelfanger *et al.* (1989) for objectiv
with interval-valued coefficients. However, the approach of Zimmerma
(1978) is used instead of the stratified piecewise reduction technique pr
posed by Rommelfanger *et al.* (1989). Although the approach propos
appears crude, it is probably sufficient for forest planning considering t
type and amount of forest information available, and the complexity
the forest ecosystem.

The FMOLP model is intuitively sound. First, the model requires on
rough estimates instead of exact values for the objective function coef
cients. Second, the model is conveniently formulated such that conve
tional solution algorithms can be used. Moreover, the capability
incorporating multiple objectives and specifying target values (appro
mated by interval limits) for each objective is appealing. Compromise s
lutions generated under this framework project fairness, particula
when dealing with a large number of decision makers typically found
forest planning environments.

The use of membership functions is one of the unique and novel fe
tures of fuzzy mathematical programming. However, it also presents o

the major limitations in incorporating imprecision and inexactness o formal optimization procedures. The form of the membership func-n used in this study is linear as described in eqn (12). While approxi-iting the membership function as linear may be justifiable in forest inning, this may not be the case for other problems where the member-p function may be more accurately represented as nonlinear. Zimmer-inn (1987) presents a number of alternative forms for the membership iction.

The MAXMIN model described in eqn (13) is one of several formula-ns that could be used under fuzzy mathematical programming. Men-za & Sprouse (1989) and Zimmermann (1987) described a number of ernative formulations depending on where and how fuzziness is re-cted in the problems (e.g. fuzziness may occur in the objective func-n/s, constraints or both; fuzziness may be reflected as fuzzy parameters coefficients).

Some observations can be noted with regards to the fuzzy approach to inning and decision making. One is the flexibility it provides in the odelling process. The classical view of optimizing the attainment of a en objective is replaced with a more practical concept of satisficing . attaining a satisfactory level of achievement). In terms of the con-aints, flexibility is reflected in treating the right-hand sides as flexible iits rather than absolute bounds.

ACKNOWLEDGEMENTS

is study was partially supported by McIntire-Stennis Research Project . 53400, Department of Forestry, Agricultural Experiment Station, iiversity of Illinois.

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